1 Prerequisites

Independence of n Variables

2 Summary

The construction of independent random variables.

3 Construction of Independent Random Variables

This is a method of constructing independent random variables $X_i$ on $([0, 1], Leb)$. We can create a random variable $X_1$ with distribution $F_1$ by using the inverse of the distribution, $X_1 = F_1^{-1}(U_1)$, where $U_1$ is a uniform[0, 1] random variable.

To use this method to generate $n$ independent random variables $X_i$ with distributions $F_i$, start with $n$ independent uniform random variables. The following is a useful method for generating any number of independent uniforms from a single uniform[0, 1] random variable. First, consider the simple case of generating two i.i.d. uniform random variables from a single uniform $U$. Consider the binary expansion of $U$,

$$U = \frac{D_1}{2} + \frac{D_2}{2^2} + \frac{D_3}{2^3} + \ldots$$

where $D_i$ is the $i^{th}$ digit in the binary expansion. Each $D_i$ takes on the value 0 or 1 with equal probability on subintervals of [0,1]. Let

$$U_1 = \frac{D_1}{2} + \frac{D_3}{2^2} + \frac{D_5}{2^3} + \ldots$$

$$U_2 = \frac{D_2}{2} + \frac{D_4}{2^2} + \frac{D_6}{2^3} + \ldots,$$
the random variables $U_1$ and $U_2$ are uniform[0, 1] and independent (a result of the fact that functions of disjoint collections of independent random variables are independent). This method can be used to generate a finite or an infinite sequence of independent uniform random variables. For an infinite sequence of random variables, consider

$$N = \bigcup_{i=1}^{\infty} N_i$$

where $|N_i| = \infty$ and the $N_i$ are disjoint. The construction above is repeated with $U_i$ defined using the digits $D_j$ where $j \in N_i$.

4 References