1 Prerequisites

Martingale convergence theorem.

2 summary

This topic states the Galton-Watson process (branching process).

3 Galton-Watson process/Critical Branching

Let $\xi^n_i, i, n \geq 0$ be i.i.d. nonnegative integer-valued random variables. Define $\{Z_n\}$, called a Galton-Watson process, by $Z_0 = 1$,

$$Z_{n+1} = \begin{cases} \xi_{1}^{n+1} + \ldots + \xi_{Z_n}^{n+1} & \text{if } Z_n > 0 \\ 0 & \text{if } Z_n = 0 \end{cases}$$

$Z_n$ represents the number of people in some population in the $n^{th}$ generation, and each person in a given generation generates some number of offspring for the next generation. Across people and generations, the number of offspring generated by a fixed person in a fixed generation are i.i.d.. If we let $\mu = E(\xi^n_i)$, then $Z_n/\mu^n$ is a martingale, but if $\mu \leq 1$, then $Z_n/\mu^n \to 0$ almost surely, even though $Z_0 = 1$.

4 Reference