1 Prerequisites

tail $\sigma$-field, $L^1$ convergence of martingales

2 Summary

This topic gives the definition and convergence of reversed Martingales.

3 Reversed Martingales

Reversed martingales arise as $\mathbb{E}(X|G_n)$ where $G_n$ is a *decreasing* rather than increasing sequence of $\sigma$—fields. Sometimes they are also called *backwards* martingales.

**Example 1** If $G_n = \sigma(X_n, X_{n+1}, \ldots)$, then $G_n \downarrow T(X_1, X_2, \ldots)$.

In general, if $G_n \downarrow G_\infty := \bigcap_n G_n$ then for $X \in L^1$ the following convergence is true in $L^1$ and a.s. sense:

$$\mathbb{E}(X|G_n) \longrightarrow \mathbb{E}(X|G_\infty).$$  \hspace{1cm} (1)

The proof is by the upcrossing inequality.

4 Reference