Key Word: Symmetry, stochastic processes
Synonym: i.i.d.

1 Prerequisite

Stochastic processes

2 Symmetry Properties

Symmetries plays a very important role in the study of stochastic processes.

Special cases of symmetries:
First take $I = \{0, 1, 2, \cdots, n\}, (X_0, X_1, \cdots, X_n)$.

- Identical Distribution: $X_i =_d X_0$ for all $i \in I$.
- Stationarity: $(X_i, X_{i+1}, \cdots, X_{i+k}) =_d (X_0, \cdots, X_k)$, for all $i, k$ such that $i \geq 0, i + k \leq n$.
- Exchangeable: $(X_{i_1}, X_{i_1+1}, \cdots, X_{i_k}) =_d (X_0, X_1, \cdots, X_k)$ for any selection of distinct indexes. Without loss of generality, can assume $k = n$.
- independent and identical distributed.
- Reversible: $(X_n, \cdots, X_0) =_d (X_0, \cdots, X_n)$.
- Markov property: $(X_0, \cdots, X_{k-1})$ is conditionally independent with $(X_k, \cdots, X_n)$ given $X_k$, for each $1 \leq k \leq n$.

Remark:
(1) i.i.d. ⇒ exchangeable⇒ stationary⇒ identical distribution; exchangeable ⇒ reversible.
(2) Say $X$ and $Z$ are **conditionally independent** give $Y$, if

$$
P(X \in A, Z \in B|Y) \overset{a.s.}{=} P(X \in A|Y)P(Z \in B|Y), \text{ for all } A \in S_X, B \in S_Z.
$$

And this is equivalent with

$$
\mathbb{E}[f(X)g(Z)|Y] \overset{a.s.}{=} \mathbb{E}[f(X)|Y]\mathbb{E}[g(Z)|Y],
$$

for all nonnegative measurable functions $f, g$.

### 3 Reference

Durrett, Rick (2005) *Probability: Theory and Examples, 3e*